

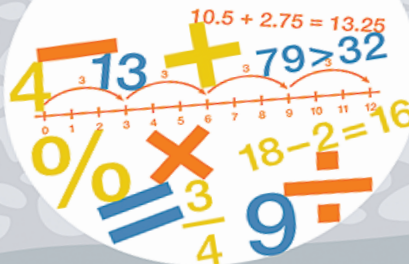
MINE THE GAP

FOR MATHEMATICAL UNDERSTANDING

Common Holes and Misconceptions
and What To Do About Them



John SanGiovanni



CM CORWIN
MATHEMATICS

GRADES 3-5

Thank you

FOR YOUR
INTEREST IN
CORWIN

Please enjoy this complimentary excerpt from Mine the Gap for Mathematical Understanding, Grades 3-5, by John SanGiovanni.

[LEARN MORE](#) about this title!

BIG IDEA 12

Connecting Multiplication and Division

TASK 12A

Write the related multiplication or division fact for each expression below.

- 7×3
- $40 \div 5$
- 2×9
- $27 \div 3$

Choose one expression above. Explain how you found the related fact. Use, pictures, numbers, or words to explain your thinking.

About the Task

Many of us see division as an unknown factor situation. Simply, we think multiplication when we see division. Yet, this can only happen when we understand the relationship and work with it in meaningful ways beyond memorizing fact families. In this task, students are prompted to create connected examples of multiplication and division expressions. They are then asked to justify one of the relationships. The products and quotients are intentionally omitted so that students cannot simply rearrange the numbers to find the inverse operation.

Anticipating Student Responses

Students may demonstrate understanding in varying ways and complexities. Some students will represent both situations with a picture or diagram establishing equal groups and groupings. Other students will show how factors and products connect with divisors and quotients. They may provide a complete fact family for the example cited. Others may extend their argument to other examples of fact families. They may write things like “what times \times equals y .” Be sure that students who rely on fact families or rearranging numbers truly understand how the two operations connect.

PAUSE AND REFLECT

- How does this task compare to tasks I've used?
- What might my students do in this task?



Visit this book's companion website at resources.corwin.com/minthegap/3-5 for complete, downloadable versions of all tasks.



WHAT THEY DID

Student 1

This student looks for a pattern within the columns. Her numbers are not written at random. For the first unknown, she explains that she found the product of 7×3 and doubled it so she could divide by 2. Her other expressions are related to either the quotient or the product in the adjacent cell. She shows some proficiency with multiplication and division but doesn't establish how the two are explicitly connected.

Student 2

This student appears to successfully connect multiplication and division expressions. She brings new, accurate information to each row. This leads us to believe she can connect multiplication and division. But does she understand why? We might ask her where or how she found the missing expressions. The prompt asks her to do that, but she communicates justification for the product of 2×9 . She doesn't explain how one expression is connected to the other.

Student 3

All of Student 3's calculations are accurate. She proves that $8 \times 5 = 40$ by giving three different representations of 8×5 to prove her factoring. These representations are striking and may quickly lead us to believe she understands the connection between the operations. Similar to Student 2, she likely understands the relationships at least between basic facts.

Student 4

This student shows clear understanding of the connection between multiplication and division. She fully explains that connection. She shares her thinking for rows 1 and 2. She uses language such as "what times five equals forty" to communicate how she found 8. She also shares how the new numbers are associated with the new expression.

USING EVIDENCE

What might we ask these students? What might we do next?

Student 1

This student's computation seems somewhat sound. The given expressions are evaluated accurately. She doubles 21 (in the first row) and 18 (third row). She has a clear understanding of multiplication and division but is unable to connect these two operations in this task. She will benefit from instructional mini-sessions that reinforce the connection. She may also benefit from matching activities (matching multiplication and division facts). After making pairs, she should be asked to explain how the two equations are connected.

Students 2 and 3

Students 2 and 3 seem to show an understanding of the relationship between multiplication and division. Their accurate calculations may lead us to think they have a better understanding of the relationship than they really do. We can simply ask how multiplication and division are related, such as the prompt in Task 12D. We may give them exact expressions and ask how the two are related. Another option is to work with

different products or factors. For example, we might ask them what multiplication expression is related to $84 \div 4 = 21$. We might ask them what division expression is related to $4 \times 16 = 64$. As students explain, we listen to identify if they just move the numbers around or if they talk about number of groups, size of groups, or how many in all. We may even need to overtly ask about those features in each equation. We might ask, "What number tells how many in all?" or "What number tells us how many are in a group?"

Student 4

Student 4 shows a developed understanding of the relationship between multiplication and division. She should be able to apply this understanding to new situations that multiply and divide with multi-digit numbers. Occasional practice with this concept will prepare her to apply the idea to expressions with one or two multi-digit factors. This will position her to be successful with partial products, partial quotients, and standard algorithms. For example, she would benefit from considering how 60×3 is related to $180 \div 3$.

TASK 12A: Write the related multiplication or division fact. Explain how you found the related fact.

Student Work 1

Use the multiplication fact to write a related division fact.

Multiplication	Division
7×3	$42 \div 2$
4×2	$40 \div 5$
2×9	$36 \div 2$
3×9	$27 \div 3 = 9$

Choose one row above. Explain how you found the related division fact. Use pictures, numbers, or words to explain your thinking.

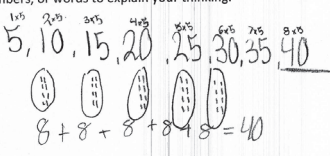
for $7 \times 3 = 21$ so I did $21 \div 2$ that = 10.5 so I did $42 \div 2$
and multiplication you just need to switch them
and that what I did for every thing else.

Student Work 3

Use the multiplication fact to write a related division fact.

Multiplication	Division
$7 \times 3 = 21$	$21 \div 7 = 3$
$8 \times 5 = 40$	$40 \div 5 = 8$
$2 \times 9 = 18$	$18 \div 2 = 9$
$9 \times 3 = 27$	$27 \div 3 = 9$

Choose one row above. Explain how you found the related division fact. Use pictures, numbers, or words to explain your thinking.

$5, 10, 15, 20, 25, 30, 35, 40$

 $8 + 8 + 8 + 8 = 40$

Student Work 2

2015

Use the multiplication fact to write a related division fact.

Multiplication	Division
7×3	$21 \div 7$
5×8	$40 \div 5$
2×9	$18 \div 9$
4×3	$27 \div 3$

Choose one row above. Explain how you found the related division fact. Use pictures, numbers, or words to explain your thinking.

I replace 9 to 10 and $10 \times 2 = 20$
Then I minus 2 so 9 will be replace
to 10.

Student Work 4

Use the multiplication fact to write a related division fact.

Multiplication	Division
7×3	$21 \div 3$
8×5	$40 \div 5$
2×9	$18 \div 2$
9×3	$27 \div 3$

Choose one row above. Explain how you found the related division fact. Use pictures, numbers, or words to explain your thinking.

for 7×3 I did $7 + 7 + 7$ and that
equaled 21, so I took the three from
the seven to make it $21 \div 3 = 7$. for $40 \div 5$
I just need to how what times five
equals forty, and that's eight, so I took
the five from forty, and we got eight times
five.

OTHER TASKS

- What will count as evidence of understanding?
- What misconceptions might you find?
- What will you do or how will you respond?



Visit this book's companion website at resources.corwin.com/minethegap/3-5 for complete, downloadable versions of all tasks.

TASK 12B: Write the missing numbers to make the statement true.

$$63 \div 7 = \underline{\hspace{2cm}} \text{ is the same as } 9 \times \underline{\hspace{2cm}} = 63.$$

$$32 \div 4 = \underline{\hspace{2cm}} \text{ is the same as } 8 \times \underline{\hspace{2cm}} = 32.$$

Explain how you found the missing numbers.

$$42 \div \underline{\hspace{2cm}} = 7 \text{ is the same as } 6 \times \underline{\hspace{2cm}} = 42.$$

$$15 \div \underline{\hspace{2cm}} = 3 \text{ is the same as } 3 \times 5 = \underline{\hspace{2cm}}.$$

Explain how you found the missing numbers.

This task is intentionally designed with complementing missing numbers (e.g., 7 is provided in the first part of the prompt and missing in the second prompt). It is designed in this way to see if students quickly see the connection and then communicate it. Do students model and diagram the situation to establish the relationship? Do other students need to rely on or even write out fact families to see the relationship between factors and divisors? Do students simply *know* the relationship between multiplication and division? In each case, students demonstrate understanding. However, there is a progression of sophistication in strategy and reasoning about these relationships.

TASK 12C: Kim knows $6 \times 5 = 30$. How can that help with $30 \div 5$? Use pictures, numbers, or words to explain your thinking.

This task provides insight into how students process the relationship between multiplication and division. As noted, some students recite fact families as the relationship between multiplication and division but are unable to extend this thinking to multi-digit number computation. Insight into their thinking, through tasks like this, helps us determine the evolution of their thinking. Consider modifying this task to make use of larger numbers. It could be rewritten as “Kim knows $6 \times 50 = 300$. How can that help her with $300 \div 50$?”

TASK 12D: How could someone prove that multiplication and division are related? Use models, numbers, or words to explain your thinking. How could it be helpful to know that multiplication and division are related?

This task is the most open of the four tasks. In some ways, it may be the best place to begin because it may provoke all sorts of reasoning. Models and equations are likely justifications. Take note of the second prompt in the task. It is open-ended with no “correct” answer. Yet, it serves as a window into how students think about and may leverage the relationship between multiplication and division. Be sure to look for students who relate that each operation is useful for working with the other.